Problem Set # 3 Solution
ECEN 3320 Fall 2013
Semiconductor Devices
September 9, 2013 – Due September 18, 2013

1. A semiconductor has $E_g = 1.40$ eV and $\mathrm{m}_h^* = 0.5m_0$; $\mathrm{m}_e^* = 0.1\mathrm{m}_0$ at $T = 300°\mathrm{K}$.

   (a) Calculate the position of the intrinsic Fermi level, E_f, with respect to the middle of the band gap, $E_i - E_{midgap} = E_i - E_g/2$.

   Solution: The intrinsic Fermi level is defined in terms of the bandgap, temperature and effective carried masses as

$$E_i - \frac{E_c - E_v}{2} = \frac{3}{4}kT\ln\left(\frac{m_h^*}{m_e^*}\right).$$

   Using the given values, we find that the distance of the Fermi level from the center of the gap is

$$E_i - \frac{E_c - E_v}{2} = \frac{3}{4}0.0258\ln(5) = 0.750.02581.61 \approx 0.0313\mathrm{eV}$$

   (b) Calculate the intrinsic carrier concentration, n_i.

   Solution: The intrinsic carrier concentration in terms of the effective densities of states, N_c and N_v and the bandgap energy as

$$n_i = \sqrt{N_c N_v}\exp\left[-E_g/kT\right].$$

   Recalling that the effecive densities of states are given by

$$\begin{aligned} N_c &= 2\left(\frac{m_e^* kT}{2\pi\hbar^2}\right)^{3/2} \\ N_v &= 2\left(\frac{m_h^* kT}{2\pi\hbar^2}\right)^{3/2} \end{aligned}$$

   we find that for a bare electron

$$N_0 = 2\left(\frac{m_0 kT}{2\pi\hbar^2}\right)^{3/2} = 2.61\times10^{19}\ \mathrm{cm}^{-3}.$$

   Using the above, we find that

$$n_i \approx (0.05)^{3/2}\,2.61\times10^{19}\times\exp(-0.70/0.0258) \approx 4.8\times10^{6}\mathrm{cm}^{-3}.$$

(c) Impurity atoms are added so that the Fermi level, E_f, is 0.35 eV above the middle of the band gap.

  i. Are the impurities donors or acceptors?

  Solution: The impurities are donors as donors raise the Fermi level. The intrinsic Fermi level is raised by only 0.03 eV relative to the center of gap. The 'measured' 0.35 eV is much larger than this indicating the semiconductor is an extrinsic n-doped semiconductor.

  ii. What was the concentration of impurities added?

  Solution: We need to make an additional assumption here, the assumption that the donor impurities are shallow. It is only the shallow impurities (within a few kT of the band edge) that will contribute to the repositioning of the Fermi level relative to the intrinsic Fermi level.

$$N_D = n_0 = n_i \exp\left[\frac{E_f - E_i}{kT}\right]$$

  that evaluates to

$$N_D \approx 1.07 \times 10^{12}\mathrm{cm}^{-3}.$$

2. Determine the equilibrium electron and hole concentrations in silicon for the following conditions.

Solution: Using the fact that there is no free charge density in an equilibrium material, we can write that

$$n_0 - p_0 + N_a^- - N_d^+ = 0.$$

The law of mass action $n_0 p_0 = n_i^2$ combined with the above gives that

$$\begin{aligned} n_0^2 + (N_a^+ - N_d^-)n_0 - n_i^2 &= 0 \\ p_0^2 + (N_d^- - N_a^+)p_0 - n_i^2 &= 0 \end{aligned}$$

with solutions (omitting the spurious one generated by multiplying through by n_0 (p_0)), we find

$$\begin{aligned} n_0 &= \left(\frac{N_d^- - N_a^+}{2}\right) + \sqrt{\left(\frac{N_d^- - N_a^+}{2}\right)^2 + n_i^2} \\ p_0 &= \left(\frac{N_a^+ - N_d^-}{2}\right) + \sqrt{\left(\frac{N_a^+ - N_d^-}{2}\right)^2 + n_i^2} \end{aligned}$$

These relations can generally be simplified significantly. For an intrinsic semiconductor, $N_d^+ \ll n_i$ and $N_a^- \ll n_i$ and, therefore, $n_0 \approx n_i + (N_d^+ - N_a^-)$ and $p_0 \approx n_i - (N_d^+ - N_a^-)$. For an extrinsic semiconductor, for example, an n-type, $n_i \ll N_d^+$ and $n_0 \approx N_d^+$ with $p_0 = n_0/n_i^2$. In our present case, the intrinsic carrier concentration is given by

$$n_i = \sqrt{N_c N_v} \exp\left[-\frac{\varepsilon_g}{2kT}\right] \approx 1 \times 10^{10}\mathrm{cm}^{-3}.$$

We see that in all cases, except (d), below, $|N_d - N_a| \gg n_i$ and, therefore, the semiconductors are extrinsic. For extrinsic, non-degenerate seminconductors, we can equate the majority concentration with the difference $|N_d - N_a|$ and calculate the minority concentration from the law of mass action that $n_i^2 = n_0 p_0$. When the semiconductor is so lightly doped that it remains intrinsic, we can simply subtract the doping concentration from the intrinsic (see (d) below).

(a) $T = 300°\text{K}$, $N_d = 2 \times 10^{15}\text{cm}^{-3}$, $N_a = 0$.
Solution: We have that

$$n_0 \approx N_d - N_a = 2 \times 10^{15}\text{cm}^{-3}$$

and therefore

$$p_0 \approx \frac{n_i^2}{n_0} \approx 5 \times 10^4\text{cm}^{-3}$$

(b) $T = 300°\text{K}$,$N_d = 0$, $N_a = 1 \times 10^{16}\text{cm}^{-3}$.
Solution: We have that

$$p_0 \approx N_a - N_d = 1 \times 10^{16}\text{cm}^{-3}$$

and therefore

$$n_0 \approx \frac{n_i^2}{p_0} \approx \times 10^4\text{cm}^{-3}$$

(c) $T = 300°\text{K}$, $N_d^+ = 2 \times 10^{15}\text{cm}^{-3}$, $N_a = 1 \times 10^{15}\text{cm}^{-3}$.
Solution: We have that

$$n_0 \approx N_d^+ - N_a^- = 1. \times 10^{15}\text{cm}^{-3}$$

and therefore

$$p_0 \approx \frac{n_i^2}{n_0} \approx \times 10^5\text{cm}^{-3}$$

(d) $T = 300°\text{K}$, $N_d^+ = 1 \times 10^9\text{cm}^{-3}$, $N_a^- = 0$.
Solution: Here we have

$$\begin{aligned} n_0 &\approx 1.1 \times 10^{10}\text{cm}^{-3} \\ p_0 &\approx 9.9 \times 10^9\text{cm}^{-3} \end{aligned}$$

(e) $T = 500°\text{K}$, $N_d^+ = 1 \times 10^{14}\text{cm}^{-3}$, $N_a^- = 0$.
Solution: At $T = 500°\text{C}$, the $n_i = N_c \exp\left[\frac{\varepsilon_c - \varepsilon_f}{kT}\right] \approx 8.53 \times 10^{13}\text{cm}^{-3}$. We need to use the general form to find the n_0 and p_0. Writing

$$n_0 = \frac{N_d^+ - N_a^+}{2} + \sqrt{\left(\frac{N_d^+ - N_a^+}{2}\right)^2 + n_i^2} \approx 1.49 \times 10^{14}\text{cm}^{-3}.$$

we therefore find that

$$p_0 \approx \frac{n_i^2}{n_0} \approx 4.89 \times 10^{13}\text{cm}^{-3}.$$

3. (a) If $E_c - E_f = 0.2\text{eV}$ in GaAs at $T = 500°\text{K}$, calculate the values of the equilibrium carrier concentrations, n and p.

Solution: The equilibrium carrier concentrations are given by

$$n \;=\; N_c(T = 500°\text{C}) \exp\left[-\frac{E_c - E_f}{kT|_{T=500°\text{C}}}\right] = 9.05 \times 10^{15}\text{cm}^{-3}.$$

The band gap of GaAs at room temperature is roughly 1.424 eV. As $E_c - E_f = 0.2\text{eV}$, then $E_f - E_v = 1.224\text{eV}$ and we can evaluate

$$p \;=\; N_v \exp\left[-\frac{E_c - E_f}{kT}\right] = 7.56 \times 10^{6}\text{cm}^{-3}.$$

(b) Assuming that the value of n you obtained in (a) remains constant, calculate $E_c - E_f$ and p at $T = 300°\text{K}$.

Solution: We have

$$E_c - E_f = -kT \ln\left(\frac{n}{N_c}\right).$$

Plugging in

$$E_c - E_f = -kT \ln\left(\frac{n = 9.05 \times 10^{15}}{N_c(500°\text{C})}\right) \approx 0.1\text{eV}.$$

The value of $E_f - E_v$ should now be circa 1.324 eV. Evaluating

$$p = N_v \exp\left(-\frac{E_f - E_v}{kT}\right) \approx 4.40 \times 10^{-4}\text{cm}^{-3}.$$

4. A certain semiconductor is doped with $N_d = 2 \times 10^{13}\text{cm}^{-3}$ and $N_A = 0$. The intrinisic carrier concentration for this semiconductor is $n_i = 2 \times 10^{13}\text{cm}^{-3}$.

(a) Determine the majority and minority carrier concentrations at thermal equilibrium.

Solution: Here, we have that $N_d^+ \approx n_i$ so we must use the general solution for concentration in terms of doping to find the effective concentrations. Writing

$$n_0 = \frac{N_d^+ - N_a^+}{2} + \sqrt{\left(\frac{N_d^+ - N_a^+}{2}\right)^2 + n_i^2} \approx 3 \times 10^{13}\text{cm}^{-3}.$$

(b) Determine the position of the energy level relative to the intrinsic Fermi level, $E_f - E_i$.

Solution: We have that

$$E_f - E_i = kT \ln\left(\frac{n}{n_i}\right) \approx 0.01\text{eV}$$